

# How many scalar fields there are and how are they related to fermions and weak bosons in the *spin-charge-family* theory?

Norma Susana Mankoč Borštnik

*Department of Physics, FMF, University of Ljubljana,  
Jadranska 19, 1000 Ljubljana, Slovenia*

## Abstract

The *spin-charge-family* theory [1–5] offers a possible explanation for the assumptions of the *standard model*, interpreting the *standard model* as its low energy effective manifestation [3]. The *standard model* Higgs and Yukawa couplings are explained as an effective replacement for several scalar fields, all of bosonic (adjoint) representations with respect to all the charge groups, with the family groups included. Assuming the Lagrange function for all scalar fields to be of the renormalizable kind, properties of the scalar fields on the tree level are discussed. Free scalar fields (mass eigen states) differ from either those, which couple to  $Z_m$ , or to  $W_m^\pm$  or to each family member of each of the four families, which further differ among themselves. Consequently the *spin-charge-family* theory predictions differ from those of the *standard model*.

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## I. INTRODUCTION

The *standard model* assumes one scalar field, the Higgs, with the charges in the fundamental representation of the charge groups and the Yukawa couplings. The Higgs, forming a kind of the "scalar Majorana", and interacting with the weak and hyper gauge bosons, determines masses of the weak bosons and, "dressing" right handed fermions with the needed weak and hyper charges, determines, together with the Yukawa couplings, also the masses of the so far observed families of fermions.

The question is: Where do the Higgs together with the Yukawa couplings of the *standard model* originate from? Is the Higgs really a scalar field with the fermionic quantum numbers in the charge sector, or it is just, so far extremely efficient, effective representation for several scalar fields which manifest as the Higgs and the Yukawa couplings?

To answer any question about the origin of the Higgs and the Yukawa couplings one first needs the answer to the question: Where do families originate? And correspondingly: How many families do we have at all?

Effective interactions can have in physics many times quite unexpected shapes and yet can be very useful (as is the case, for example, with by the experiments suggested spin-spin interactions in several models describing (anti)feroelectric and (anti)ferromagnetic materials and also other properties, where the interaction of the electromagnetic origin among many electrons and nuclei involved can effectively be expressed by a kind of the "spin-spin" interaction).

I am proposing the theory [1–7], the *spin-charge-family* theory, which does offer the explanation for the origin of families, of vector gauge fields and of several scalar fields: A simple starting action at higher dimensions determines at low energies properties of families of fermions, of the known vector gauge fields and of several scalar fields. Vector and scalar fields, which originate in the spin connections and vielbeins at higher dimensions, have correspondingly the charges in the adjoint representations with respect to all charge groups. Families appear in the theory due to the fact that there are two kinds of gamma matrices (two kinds of the Clifford algebra objects, only two): i. The one used by Dirac to describe the spin of fermions (spinors). ii. The second one used in this theory to explain the origin of families [13]. The theory predicts before the electroweak break four families of ( $u^i$  and  $d^i$ ,  $i \in \{1, 2, 3, 4\}$ ) quarks and ( $e^i$  and  $\nu^i$ ) leptons, left handed weak charged and right

handed weak chargeless, which when coupling to massive scalar fields with non zero vacuum expectation values and to gauge fields, become massive.

In this paper the scalar fields and their taking care of the masses of quarks and leptons and of the weak bosons on the tree level are studied and the predictions discussed. Although the *spin-charge-family* theory still requires (many) additional studies to be proved – or disproved – that it is the right step beyond the *standard model*, yet the work done so far [1–8] gives a hope.

Keeping the symmetries of four massless families (the spins and charges of quarks and leptons, left and right handed) and of the massless gauge fields and the scalar fields as they follow from the *spin-charge-family* theory before the electroweak break and letting the scalar fields to gain nonzero vacuum expectation values, the tree level contributions strongly relate properties of family members and also of the gauge weak fields. It is a hope, supported by the calculations done so far [4, 6], that loop corrections (in all orders) lead to the observed properties of fermions.

In this paper I assume (not derive from the *spin-charge-family* theory) the Lagrange function for the scalar fields, which cause the electroweak break, and their couplings to gauge bosons (Eq.(7)) so that the theory is renormalizable. This assumption is made only to manifest that the existence of several scalars might strongly influence the experiments, which search for the Higgs.

After the electroweak break the effective Lagrange density for the four families of fermions looks in the *spin-charge-family* theory [3] as

$$\begin{aligned}
\mathcal{L}_f &= \bar{\psi} (\gamma^m p_{0m} - M) \psi, \\
p_{0m} &= p_m - \{g^Y \cos \theta Q A_m + g^1 \cos \theta Q' Z_m^{Q'} + \frac{g^1}{\sqrt{2}} (\tau^{1+} W_m^{1+} + \tau^{1-} W_m^{1-}) + g^{Y'} \cos \theta' Y' A_m^{Y'}\} \\
\bar{\psi} M \psi &= \bar{\psi} \gamma^s p_{0s} \psi, \quad s \in \{7, 8\}, \\
p_{0s} &= p_s - \{\tilde{g}^{\tilde{N}_L} \vec{\tilde{N}}_L \vec{\tilde{A}}_s^{\tilde{N}_L} + \tilde{g}^{\tilde{Q}'} \tilde{Q}' \tilde{A}_s^{\tilde{Q}'} + \frac{\tilde{g}^1}{\sqrt{2}} (\tilde{\tau}^{1+} \tilde{A}_s^{1+} + \tilde{\tau}^{1-} \tilde{A}_s^{1-}) \\
&\quad + g_1^Y \cos \vartheta_1 Q A_s^Q + g_1^1 \cos \vartheta_1 Q' Z_s^{Q'} + \frac{g^1}{\sqrt{2}} (\tau^{1+} W_s^{1+} + \tau^{1-} W_s^{1-}) \\
&\quad + g^2 \cos \vartheta_2 Y' A_s^{Y'}\}, \quad Q = \tau^{13} + Y, \quad Q' = (\tau^{13} - \tan^2 \vartheta_1 Y), \quad \frac{g_1^Y}{g_1^1} = \tan \vartheta_1. \quad (1)
\end{aligned}$$

$Q$  and  $Q'$  are the *standard model* like charges ( $\vartheta_1$  does not need to be  $\theta$ ), while  $Y'$  is the additional one [3], appearing in the *spin-charge-family* theory after the break which is followed by the electroweak break.

Taking into account that  $(\pm) = \frac{1}{2}(\gamma^7 \pm \gamma^8)$ , the mass term  $\bar{\psi} M \psi$  of the Lagrange function  $\mathcal{L}_f$  can be rewritten as follows

$$\bar{\psi} M \psi = \bar{\psi} \begin{pmatrix} 78 \\ - \end{pmatrix} p_{0-} + \begin{pmatrix} 78 \\ + \end{pmatrix} p_{0+} \psi, \quad p_{0\mp} = (p_{07} \pm ip_{08}). \quad (2)$$

The mass term  $\bar{\psi} M \psi$  determines the tree level mass matrices of quarks and leptons and correspondingly the masses, the Yukawa couplings and the mixing matrices for the four families after the loop corrections are taken into account. The scalar fields – the triplets  $(\vec{A}_{\mp}^{\tilde{N}_L}, \vec{A}_{\mp}^{\tilde{1}}, \vec{A}_{\mp}^1, \vec{A}_{\mp}^o)$  and the singlets  $(A_{\mp}^{Y'}, A_{\mp}^Y)$  – are with their vacuum expectation values responsible for the appearance of the fermionic masses [14].

The existence of the two scalar triplets,  $\vec{A}_{\mp}^1$  and  $\vec{A}_{\mp}^o$ , with the properties

$$\begin{aligned} Y \vec{A}_{\mp}^1 &= -\vec{A}_{\mp}^1, \quad T^{13} \vec{A}_{\mp}^1 = (A_{\mp}^{11}, 0, -A_{\mp}^{12}), \quad \langle \vec{A}_{\mp}^1 \rangle = (v_{11\mp}, 0, 0), \\ Y \vec{A}_{\mp}^o &= 0, \quad T^{13} \vec{A}_{\mp}^o = (A_{\mp}^{o1}, 0, -A_{\mp}^{o2}), \quad \langle \vec{A}_{\mp}^o \rangle = (0, v_{o3\mp}, 0), \end{aligned} \quad (3)$$

where  $\langle \vec{A}_{\mp}^1 \rangle$  and  $\langle \vec{A}_{\mp}^o \rangle$  denote the vacuum expectation values, are assumed. Both contribute to the masses of the weak bosons on the tree level. Although all the scalar fields have in the *spin-charge-family* theory charges in the adjoint representation, it can not be expected that a triplet is a good replacement for several scalar fields – vielbeins and spin connections of the two kinds – involved in the electroweak break. I also take into account only the tree level contributions, although it might turn out that the loop corrections are very important and are needed probably in all orders to reproduce the measured properties of fermions and weak bosons.

The tree level mass term  $\bar{\psi} M \psi$  distinguishes among the members of any of the four families – due to the operators  $Q$  and  $Q'$  and due to the operators  $\begin{pmatrix} 78 \\ \mp \end{pmatrix}$ , which transform the weak and hyper charges of  $u_R^i$ -quarks and  $\nu_R^i$ -leptons  $\begin{pmatrix} 78 \\ (-) \end{pmatrix}$  into those of the corresponding left handed ones and the weak and hyper charges of  $d_R^i$ -quarks and  $e_R^i$ -leptons  $\begin{pmatrix} 78 \\ (+) \end{pmatrix}$  into those of their left handed ones [3]. The reader can find a short explanation in appendix . The fields  $\vec{A}_{\mp}^{\tilde{N}_L^i}$  and  $\vec{A}_{\mp}^{1i}$ , the two kinds of triplets, couple before the electroweak break to the family members due to the operators  $\vec{\tau}^1$  and  $\vec{N}_L$  as demonstrated in the diagram below

$$\begin{pmatrix} III & IV \\ I & II \end{pmatrix} \begin{matrix} \xleftrightarrow{\vec{N}_L} \\ \updownarrow \vec{\tau}^1 \end{matrix}. \quad (4)$$

The numbers in the diagram determine the four massless families before the electroweak break, presented in Table II of appendix . All the scalar fields are in the adjoint representation of the charge groups, they are either triplets or singlets.

Let us denote the scalar fields – triplets and singlets – which in the electroweak break gain nonzero vacuum expectation values (all in pairs  $(\mp)$ ), with a common vector

$$\begin{aligned}\Phi^{Ai} &\equiv \Phi_{\mp}^{Ai} = (\vec{A}_{\mp}^{\tilde{N}_L}, \vec{A}_{\mp}^1, \vec{A}_{\mp}^1, \vec{A}_{\mp}^o, A_{\mp}^Y, A_{\mp}^{Y'}), \\ \Phi_{\mp}^{Ai} &= (\Phi_7^{Ai} \pm i\Phi_8^{Ai}), \quad A = \{\tilde{N}_L, \tilde{1}, 1, o, Y, Y'\},\end{aligned}\tag{5}$$

and let us assume that the effective potential  $V(\Phi^{Ai})$ , coupling all the (assumed to be real) scalar fields, is renormalizable

$$V(\Phi^{Ai}) = \sum_{A,i} \left\{ -\frac{1}{2} (m_{Ai})^2 (\Phi^{Ai})^2 + \frac{1}{4} \sum_{B,j} \lambda^{Ai Bj} (\Phi^{Ai})^2 (\Phi^{Bj})^2 \right\}.\tag{6}$$

Couplings among the scalar fields are here chosen to be symmetric:  $\lambda^{Ai Bj} = \lambda^{Bj Ai}$ . The scalar triplets  $\vec{A}_{\mp}^1$  and  $\vec{A}_{\mp}^o$  couple to the gauge bosons according to the Lagrange function  $\mathcal{L}_s$

$$\begin{aligned}\mathcal{L}_s &= \sum_{A,i} (p_{0m} \Phi^{Ai})^\dagger (p_0^m \Phi^{Ai}) - V(\Phi^{Ai}), \\ p_{0m} &= p_m - \{g^Y \ Y \ A_m^Y + g^1 \ \vec{T}^1 \ \vec{A}_m^1\}.\end{aligned}\tag{7}$$

All the components of  $\vec{A}_{\mp}^1$  and  $\vec{A}_{\mp}^{\tilde{N}_L}$  and also of  $A_{\mp}^Q$  and  $Z_{\mp}^{Q'}$ ,

$$\begin{aligned}A_{\mp}^Y &= -\sin \vartheta_1 Z_{\mp}^{Q'} + \cos \vartheta_1 A_{\mp}^Q, \\ A_{\mp}^{13} &= \cos \vartheta_1 Z_{\mp}^{Q'} + \sin \vartheta_1 A_{\mp}^Q,\end{aligned}\tag{8}$$

are assumed to have after the electroweak break nonzero vacuum expectation values, except the triplets  $\vec{A}_{\mp}^1$  and  $\vec{A}_{\mp}^o$ , for which we assume, in order that the electromagnetic field stay massless, that only the components  $A_{\mp}^{11}$  and  $A_{\mp}^{o3}$  have nonzero vacuum expectation values (Eq.(3)),  $\langle \vec{A}_{\mp}^1 \rangle = (v_{11\mp}, 0, 0)$  and  $\langle \vec{A}_{\mp}^o \rangle = (0, v_{o3\mp}, 0)$ , so that  $Q \langle \vec{A}_{\mp}^1 \rangle = 0$  and  $Q \langle \vec{A}_{\mp}^o \rangle = 0$ . This choice relates the vacuum expectation values of  $Z_{\mp}^{Q'}$  and  $A_{\mp}^Q$  ( $v_{Q'\mp}$  and  $v_{Q\mp}$  respectively)

$$\frac{v_{Q\mp}}{v_{Q'\mp}} = -\tan^{-1} \vartheta_1, \quad \frac{v_{Y\mp}}{v_{Q'\mp}} = -\sin^{-1} \vartheta_1.\tag{9}$$

## II. SUPERPOSITION OF SCALAR FIELDS IN MINIMIZATION PROCEDURE ON TREE LEVEL

Let us look for the minimum of the potential of Eq. (6) and search for the mass eigenstates on the tree level. First we find the first derivatives with respect to all the scalar fields and put them equal to zero

$$\frac{\partial V(\Phi^{Ai})}{\partial \Phi^{Ai}} = 0 = \Phi^{Ai} [-(m_{Ai})^2 + \lambda^{Ai}(\Phi^{Ai})^2 + \sum_{B,j} \lambda^{Ai Bj} (\Phi^{Bj})^2]. \quad (10)$$

Here the notation  $\lambda^{Ai} := \lambda^{Ai Ai}$  is used. When expressing the minimal values of the scalar fields, let us call them  $v_{Ai}$ , as functions of the parameters, Eq. (10) leads to the coupled equations for the same number of unknowns  $v_{Ai} = \Phi_{min}^{Ai}$

$$-(m_{Ai})^2 + \sum_{B,j} \lambda^{Ai Bj} (v_{Bj})^2 = 0. \quad (11)$$

Looking for the second derivatives at the minimum determined by  $v_{Ai}$  one finds

$$\frac{\partial^2 V(\Phi^{Ck})}{\partial \Phi^{Ai} \partial \Phi^{Bj}} \Big|_{v_{Ck}} = 2\lambda^{Ai Bj} v_{Ai} v_{Bj}. \quad (12)$$

Let us look for the basis  $\Phi^\beta$  (we should not forget the index  $(\pm)$ ),

$$\Phi^{Ai} = \sum_{\beta} \mathcal{C}_{\beta}^{Ai} \Phi^{\beta}, \quad (13)$$

in which on the tree level the potential would be diagonal

$$V(\Phi^{\beta}) = \sum_{\beta} \left\{ -\frac{1}{2} (m_{\beta})^2 (\Phi^{\beta})^2 + \frac{1}{4} \lambda^{\beta} (\Phi^{\beta})^4 \right\}, \quad (14)$$

with  $\frac{\partial V}{\partial \Phi^{\beta}} \Big|_{v_{Ai}} = \sum_{Ai} \frac{\partial V}{\partial \Phi^{Ai}} \frac{\partial \Phi^{Ai}}{\partial \Phi^{\beta}} \Big|_{v_{Ai}} = 0$ , with  $\Phi_{min}^{\beta}$  at these points called  $v_{\beta} = \sum_{Ai} \mathcal{C}_{\beta}^{Ai T} v_{Ai}$  ( $T$  denotes transposition), and correspondingly with  $\frac{\partial^2 V}{\partial (\Phi^{\beta})^2} \Big|_{v_{\beta}} = -(m_{\beta})^2 + 3\lambda^{\beta} (\Phi^{\beta})^2 \Big|_{v_{\beta}} = 2\lambda^{\beta} (v_{\beta})^2 = \sum_{A,i,B,k} 2\lambda^{Ai Bj} v_{Ai} v_{Bj} \mathcal{C}_{\beta}^{Ai} \mathcal{C}_{\beta}^{Bj}$ . This means that the new basis can be found by diagonalizing the matrix of the second derivatives at the minimum and correspondingly put to zero the determinant

$$\det \begin{pmatrix} 2\lambda^{\tilde{N}_L 1} (v_{\tilde{N}_L 1})^2 - 2(m_{\beta})^2, & 2\lambda^{\tilde{N}_L 1 \tilde{N}_L 2} v_{\tilde{N}_L 1} v_{\tilde{N}_L 2}, & 2\lambda^{\tilde{N}_L 1 \tilde{N}_L 3} v_{\tilde{N}_L 1} v_{\tilde{N}_L 3}, & \dots \\ 2\lambda^{\tilde{N}_L 2 \tilde{N}_L 1} v_{\tilde{N}_L 2} v_{\tilde{N}_L 1}, & 2\lambda^{\tilde{N}_L 2} (v_{\tilde{N}_L 2})^2 - 2(m_{\beta})^2, & 2\lambda^{\tilde{N}_L 2 \tilde{N}_L 3} v_{\tilde{N}_L 2} v_{\tilde{N}_L 3}, & \dots \\ \vdots & \vdots & \ddots & \ddots \end{pmatrix}. \quad (15)$$

The same number of orthogonal scalar fields  $\Phi^\beta$ , with nonzero vacuum expectation values and nonzero masses, as we started with, follow. To each of them one eigen value  $2(m_\beta)^2$  corresponds, determined by the parameters  $m_{Ai}$  and  $\lambda^{AiBj}$  of Eq. (6).

For the time evolution of the free scalar fields one correspondingly finds for each  $\beta$

$$\Phi^\beta(t) = e^{-im_\beta(t-t_0)} \Phi^\beta(t_0). \quad (16)$$

### A. A simple example

Let us examine a simple case, one triplet, say  $\vec{A}^1$ , and let us call these three scalar states  $\Phi^i$ . Following Eq. (10) one obtains  $-(m_i)^2 + \sum_j \lambda^{ij}(v_j)^2 = 0$ , for each  $i = 1, 2, 3$ . Let us further simplify the example by the assumption that one of these three fields is decoupled:  $\lambda^{i3} = 0$ , for  $i = (1, 2)$ . Then it follows for the vacuum expectation values  $v_i, i \in \{1, 2, 3\}$

$$(v_1)^2 = \frac{-\lambda^{12}(m_2)^2 + \lambda^2(m_1)^2}{\lambda^1\lambda^2 - (\lambda^{12})^2}, \quad (v_2)^2 = \frac{-\lambda^{12}(m_1)^2 + \lambda^1(m_2)^2}{\lambda^1\lambda^2 - (\lambda^{12})^2}, \quad (v_3)^2 = \frac{(m_3)^2}{\lambda^3}. \quad (17)$$

The second derivatives at the minimum,  $\frac{\partial^2 V(\Phi^k)}{\partial \Phi^i \partial \Phi^j} |_{\Phi^k=v_k} = 2\lambda^{ij}v_i v_j$ , lead to the determinant (Eq.(15)), from where one obtains the eigen masses

$$(m^{1,2})^2 = \frac{1}{2} \{ [\lambda^1(v_1)^2 + \lambda^2(v_2)^2] \mp \sqrt{[\lambda^2(v_2)^2 - \lambda^1(v_1)^2]^2 + 4(\lambda^{12})^2 (v_1)^2 (v_2)^2} \}, \quad (18)$$

and  $(m^3)^2 = (m_3)^2$ . If the coupling between the two scalar components is zero, the trivial case of three uncoupled scalar fields follows. In the case that the two masses,  $m_1$  and  $m_2$ , are equal and that also the two self strengths are the same,  $\lambda^1 = \lambda^2$ , then  $(v_1)^2 = (v_2)^2$  and the two eigen values for masses are  $(m^{1,2})^2 = (v_1)^2 [(\lambda^1 - \lambda^{12}), (\lambda^1 + \lambda^{12})]$ . In the case that  $\lambda^1$  and  $\lambda^{12}$  are close to each other, the two eigen values differ a lot. In the case of  $\lambda^{12} = 0$  the two scalars would manifest as only one.

Such a simplified situation illustrates that the mass eigen states of the scalar fields might differ a lot from the superposition of the scalar fields which couples to any of the family members of any of the families, the tree level mass matrices of which are presented in Table I and in Eq. (19) and discussed in next section III.

### III. COUPLING OF FAMILY MEMBERS TO SCALAR FIELDS

The tree level contributions of  $\vec{A}_{\mp}^{\tilde{N}_L}$  and  $\vec{A}_{\mp}^1$  (Eq. (1)) to the mass matrix of any family member look [3, 6] as it is presented in Table I. The notation  $\tilde{a}_{\mp}^{\tilde{A}i} = -\tilde{g}^{\tilde{A}} \tilde{v}_{\tilde{A}i\mp}$  is used,

$i$	1	2	3	4
1	$-\frac{1}{2}(\tilde{a}_{\mp}^{13} + \tilde{a}_{\mp}^{\tilde{N}_L^3})$	$\tilde{a}_{\mp}^{\tilde{N}_L^-}$	0	$\tilde{a}_{\mp}^{1-}$
2	$\tilde{a}_{\mp}^{\tilde{N}_L^+}$	$\frac{1}{2}(-\tilde{a}_{\mp}^{13} + \tilde{a}_{\mp}^{\tilde{N}_L^3})$	$\tilde{a}_{\mp}^{1-}$	0
3	0	$\tilde{a}_{\mp}^{1+}$	$\frac{1}{2}(\tilde{a}_{\mp}^{13} - \tilde{a}_{\mp}^{\tilde{N}_L^3})$	$\tilde{a}_{\mp}^{\tilde{N}_L^-}$
4	$\tilde{a}_{\mp}^{1+}$	0	$\tilde{a}_{\mp}^{\tilde{N}_L^+}$	$\frac{1}{2}(\tilde{a}_{\mp}^{13} + \tilde{a}_{\mp}^{\tilde{N}_L^3})$

TABLE I: The contributions of the fields  $(-\tilde{g}^1 \tilde{\tau}^1 \tilde{A}_{\mp}^1, -\tilde{g}^{\tilde{N}_L} \tilde{N}_L^i \tilde{A}_{\mp}^{\tilde{N}_L})$  to the mass matrices on the tree level ( $\mathcal{M}_{(o)}$ ) for the lower four families of quarks and leptons after the electroweak break are presented. The notation  $\tilde{a}_{\mp}^{\tilde{A}i} = -\tilde{g}^{\tilde{A}} \tilde{v}_{\mp}^{\tilde{A}i}$  is used.

where  $\tilde{v}_{\tilde{A}i\mp}$  are the vacuum expectation values of the corresponding scalars. Let us repeat that  $\tilde{a}_{\mp}^{\tilde{A}i}$  distinguish among  $(u^i, \nu^i)$   $((-))$  and  $(d^i, e^i)$   $((+))$ . Since  $\tau^{1i}$  on the right handed spinors give zero, the triplets  $\vec{A}_{\mp}^1$ , as well as  $\vec{A}_{\mp}^0$  do not contribute to the fermion masses. The contributions of  $g_1^Y \cos \vartheta_1 Q A_{\mp}^Q$ ,  $g_1^1 \cos \vartheta_1 Q' Z_{\mp}^{Q'}$ , and  $g^{Y'} Y' A_{\mp}^{Y'}$  are not presented in Table I. They are different for each of the family member  $\alpha = (u^i, d^i, \nu^i, e^i)$  and the same for all the families ( $i = (1, 2, 3, 4)$ )

$$a_{\mp}^{\alpha} = -\{g_1^Q Q^{\alpha} v_{Q\mp} + g_1^{Q'} Q'^{\alpha} v_{Q'\mp} + g^{Y'} Y'^{\alpha} v_{Y'\mp}\}, \quad (19)$$

with  $Q^{\alpha}$ ,  $Q'^{\alpha}$  and  $Y'^{\alpha}$ , which are eigen values of the corresponding operators for the spinors state  $\alpha$ . When assuming that the triplets of Eq. (3) determine masses of the weak bosons on the tree level, Eq. (9) relates the vacuum expectation values  $v_{Q\mp}$ ,  $v_{Q'\mp}$  and  $\tan \vartheta_1$ . Correspondingly we have  $a_{\mp}^{\alpha} = \frac{g_1^Y}{\sin \vartheta_1} Y^{\alpha} v_{Q'\mp} - g^{Y'} Y'^{\alpha} v_{Y'\mp}$ .

Also possible Majorana term, appearing in the theory, and manifesting in higher orders, is not in the table of the tree level contributions. Loop corrections, to which also the massive gauge fields and dynamical massive scalar fields contribute, are expected to strongly influence fermions properties. These calculations are in progress [6] and look so far promising in offering the right answers for the masses and mixing matrices of fermions.

Let  $\psi_{(L,R)}^{\alpha}$  denote massless and  $\Psi_{(L,R)}^{\alpha}$  massive four vectors for each family member  $\alpha = (u_{L,R}, d_{L,R}, \nu_{L,R}, e_{L,R})$  after taking into account loop corrections in all orders [3, 6],  $\psi_{(L,R)}^{\alpha} = V_{(L,R)}^{\alpha} \Psi_{(L,R)}^{\alpha}$ , and let  $(\psi_{(L,R)}^{\alpha k}, \Psi_{(L,R)}^{\alpha k})$  be any component of the four vectors, massless and



massive, respectively. On the tree level we have  $\psi_{(L,R)}^\alpha = V_{(o)}^\alpha \Psi_{(L,R)}^{\alpha(o)}$  and

$$\langle \psi_L^\alpha | \gamma^0 \mathcal{M}_{(o)}^\alpha | \psi_R^\alpha \rangle = \langle \Psi_L^{\alpha(o)} | \gamma^0 V_{(o)}^{\alpha\dagger} \mathcal{M}_{(o)}^\alpha V_{(o)}^\alpha | \Psi_{R(o)}^\alpha \rangle, \quad (20)$$

with  $\mathcal{M}_{(o)kk'}^\alpha = \sum_{A,i} (-g^{Ai} v_{Ai\mp}) C_{kk'}^\alpha$ . The coefficients  $C_{kk'}^\alpha$  can be read from Table I. It then follows

$$\begin{aligned} \bar{\Psi}^\alpha V_{(o)}^{\alpha\dagger} \mathcal{M}_{(o)}^\alpha V_{(o)}^\alpha \Psi^\alpha &= \bar{\Psi}^\alpha \text{diag}(m_{(o)1}^\alpha, \dots, m_{(o)4}^\alpha) \Psi^\alpha, \\ V_{(o)}^{\alpha\dagger} \mathcal{M}_{(o)}^\alpha V_{(o)}^\alpha &= \Phi_{f(o)}^\alpha. \end{aligned} \quad (21)$$

The coupling constants  $m_{(o)k}^\alpha$  (in some units) of the dynamical scalar fields  $\Phi_{f(o)k}^\alpha$  to the family member  $\Psi^{\alpha k}$  belonging to the  $k^{th}$  family are on the tree level correspondingly equal to

$$(\Phi_{\Psi(o)}^\alpha)_{kk'} \Psi^{\alpha k'} = \delta_{kk'} m_{(o)k}^\alpha \Psi^{\alpha k}. \quad (22)$$

The superposition of scalar fields  $(\Phi_{f(o)}^\alpha)$ , which couple to fermions [15] and depend on the quantum numbers  $\alpha$  and  $k$ , are in general different from the superposition  $\Phi^\beta$  (Eqs. (13,16)), which are the mass eigen states. Each family member  $\alpha$  of each massive family  $k$  couples in general to different superposition of scalar fields.

The two kinds of superposition are expressible with each other

$$\Phi_{f(o)k}^\alpha = \sum_\beta D_k^{\alpha\beta} \Phi^\beta. \quad (23)$$

#### IV. SCALAR TRIPLETS BRING MASSES TO WEAK BOSONS

According to the assumption of Eqs.(7) and (3) there are triplets  $\vec{A}_\mp^1$  and  $\vec{A}_\mp^o$  with their nonzero vacuum expectation values, which manifest after the electroweak phase transition as a replacement for vielbeins and those spin connection fields which couple to the weak boson gauge field  $Z_m^{Q'}$  and  $W_m^{1\pm}$ .

This replacement might not be the right one, yet it is useful to demonstrate that scalar fields to which the weak bosons couple might differ a lot from the mass eigen states as well as from those to which fermions couple.

The operators  $(-g^1 \vec{T} \vec{A}_m^1 - g^Y Y A_m^Y)$  operating on the scalar triplets  $(\vec{A}_\mp^1, \vec{A}_\mp^o)$  form the

$3 \times 3$  matrix

$$\begin{pmatrix} -g^1 A_m^{13} + g^Y A_m^Y & -g^1 \frac{1}{\sqrt{2}} (A_m^{11} - iA_m^{12}) & 0 \\ -g^1 \frac{1}{\sqrt{2}} (A_m^{11} + iA_m^{12}) & g^Y A_m^Y & -g^1 \frac{1}{\sqrt{2}} (A_m^{11} - iA_m^{12}) \\ 0 & -g^1 \frac{1}{\sqrt{2}} (A_m^{11} + iA_m^{12}) & g^1 A_m^{13} + g^Y A_m^Y \end{pmatrix}. \quad (24)$$

Assuming a new superposition of gauge fields as in the *standard model*

$$A_m^{13} = \cos \theta Z_m^{Q'} + \sin \theta A_m^Q, \quad A_m^Y = -\sin \theta Z_m^{Q'} + \cos \theta A_m^Q, \quad W_m^{1\pm} = \frac{1}{\sqrt{2}} (A_m^{11} \mp iA_m^{12}), \quad (25)$$

we end up with the matrix which, when being applied on  $(v_{11\mp}, 0, 0)$ , leads to the three vector  $(-\frac{g^1}{\cos \theta} v_{11\mp} Z_m^{Q'}, -g^1 v_{11\mp} W_m^{1-}, 0)$ . The operators  $(-g^1 \vec{T} \vec{A}_m^1 - g^Y Y A_m^Y)$  operating on the scalar triplet  $(0, v_{o3\mp}, 0)$  gives the three vector  $(v_{o3\mp} W_m^{1+}, 0, v_{o3\mp} W_m^{1-})$ .

The contribution of both triplets to the masses of the weak bosons on the tree level is correspondingly

$$(g^1)^2 |v_{11\mp}|^2 \left( \frac{1}{(\cos \theta)^2} Z_m^{Q'} Z_m^{Q'm} + (1 + 2 \left| \frac{v_{o3\mp}}{v_{11\mp}} \right|^2) W_m^{1+} W_m^{1-m} \right). \quad (26)$$

The *standard model* weak bosons mass term obtained on a tree level by the Higgs, which is a weak doublet, carries the hyper charge  $Y = \frac{1}{2}$  and has the vacuum expectation value  $(0, v)$ , is equal to  $(\frac{1}{2})^2 (g^1)^2 v^2 (\frac{1}{(\cos \theta_W)^2} Z_m Z_m^m + 2 W_m^+ W_m^-)$ . The triplet  $\vec{A}_\mp^1$  gives for the factor of  $\sqrt{2}$  too large mass of  $Z^m$  with respect to the mass of  $W^{\pm m}$ . If the triplet  $\vec{A}_\mp^o$  contributes so that  $|\frac{g^0 v_{o3\mp}}{g^1 v_{11\mp}}|^2 = \frac{1}{2}$ , the sum of both contributions give the weak boson mass ratio on the tree level as the *standard model* does.

Since the Lagrange function (Eq. (7)) is assumed (from the symmetry requirements), not derived from the starting Lagrange function for the vielbeins and the two kinds of the spin connection fields, the result is not trustable, but meaningful, if the requirement that  $|\frac{g^0 v_{o3\mp}}{g^1 v_{11\mp}}|^2 = \frac{1}{2}$  can be proved. One can also hardly expect that the tree level contributions to the ratio of the weak bosons masses is in a good agreement with the measured ones, while the tree level mass matrices for fermions, when diagonalized, do not fit well to the experimental masses and mixing matrices. Since the so far done calculations show [6] that loop corrections might lead to the mass matrices which result in the measured properties of fermions, loop corrections to the weak boson masses might influence the ratio of boson masses as well [16].

Let us conclude this section by looking at the time evolution of the two scalar triplets  $\vec{A}_{\mp}^k$ ,  $k = 1, o$  (Eqs. (13, 16))

$$\Phi^{ki}(t, t_0) = \sum_{\beta} \mathcal{C}_{\beta}^{ki} e^{-im_{\beta}(t-t_0)} \Phi^{\beta}, \quad k = 1, o. \quad (27)$$

## V. CONCLUSIONS

It is demonstrated (on the tree level only) that according to the *spin-charge-family* theory – which predicts the families of fermions and their charges, the gauge fields and several scalar fields – each family member ( $\alpha = (u, d, \nu, e)$ )  $\Psi^{\alpha k}$  of each family ( $k = (1, 2, 3, 4)$ ) couples to a different superposition of the scalar fields ( $\Phi_{f(0)}^{\alpha}$ ), Eq. (21), with the coupling constant proportional to its (fermion) mass (Eq. (22)). Each of these superposition differs from the scalar fields (Eq. (27)), assumed to be triplets (Eqs. (3), (7)), which contribute to the masses of the weak gauge bosons (Eq. (7)). The scalar mass eigen states ( $\Phi^{\beta}$ ) form the superposition (Eqs.(13), (14)), which again differ from all the above mentioned superposition of the scalar fields. Properties depend on the parameters, the values of which are in this paper not discussed.

Although the ratio of the masses of the weak gauge bosons  $\frac{m_W}{m_Z}$ , determined by the assumed (not derived from the starting action) triplets on the tree level, does agree with the *standard model* prediction ( $\frac{m_W}{m_Z} = \frac{\cos\theta_1}{\sqrt{2}}$ ) on the tree level under the condition that the ratio  $(\frac{v_{o3\mp}}{v_{11\mp}})^2 = \frac{1}{2}$ , (Eqs. (26)), the loop corrections might drastically change the results for fermions [6], weak bosons and scalars properties. Yet the analyses clearly shows, that several scalar fields can hardly be seen in all the experiments as only one Higgs as predicted by the *standard model*.

It appears as a great challenge to explain, if the *standard model* is really a low energy effective manifestation of the *spin-charge-family* theory (or of any other theory which is able to explain the existence of the families and correspondingly of several scalar fields leading to mass matrices and Yukawa couplings), why the *standard model*, with the scalar fields replaced by a weak doublet (with the charges in the fundamental representation), does predict the mass ratio already on the tree level in such a good agreement with the experimental data (although with the experimentally obtained fermion properties which take into account to some extent all loop corrections). Since taking into account loop corrections in all orders

manifests a complicated many body problem, such explanation might be very difficult as we learn from several very efficient effective theories in many body problems.

Let me conclude with the predictions of the *spin-charge-family* theory on the tree level: Observations of the scalar fields at the LHC and other experiments might differ a lot from the predictions of the *standard model*, although so far the experimental data have shown no disagreement with the *standard model* predictions. A systematic study of predictions of the *spin-charge-family* theory is needed. The predictions for the observation of the scalar fields is in progress [11].

### Appendix: The technique for representing spinors [1, 3, 9, 10]

The technique [1, 3, 9, 10] can be used to construct a spinor basis for any dimension  $d$  and any signature in an easy and transparent way. Equipped with the graphic presentation of basic states, the technique offers an elegant way to see all the quantum numbers of states with respect to the two Lorentz groups with the infinitesimal generators of the groups  $S^{ab}$  and  $\tilde{S}^{ab}$ , as well as transformation properties of the states under any Clifford algebra object  $\gamma^a$  and  $\tilde{\gamma}^a$ ,  $\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab}$ ,  $\{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}$ ,  $\{\gamma^a, \tilde{\gamma}^b\}_+ = 0$ , for any  $d$ , even or odd.

Since the Clifford algebra objects  $S^{ab} = (i/4)(\gamma^a\gamma^b - \gamma^b\gamma^a)$  and  $\tilde{S}^{ab} = (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$  close the algebra of the Lorentz group, while  $\{S^{ab}, \tilde{S}^{cd}\}_- = 0$ ,  $S^{ab}$  and  $\tilde{S}^{ab}$  form the equivalent representations to each other. If  $S^{ab}$  are used to determine spinor representations in  $d$  dimensional space, and after the break of symmetries, the spin and the charges in  $d = (1+3)$ , can  $\tilde{S}^{ab}$  be used to describe families of spinors.

To make the technique simple the graphic presentation of nilpotents and projectors was introduced [10]. For even  $d$  we have

$$\begin{aligned} \begin{matrix} ab \\ (k) \end{matrix} &:= \frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b), & \begin{matrix} ab \\ [k] \end{matrix} &:= \frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b), \end{aligned} \quad (\text{A.1})$$

with the properties  $k^2 = \eta^{aa}\eta^{bb}$  and

$$S^{ab} \begin{matrix} ab \\ (k) \end{matrix} = \frac{1}{2} k \begin{matrix} ab \\ (k) \end{matrix}, \quad S^{ab} \begin{matrix} ab \\ [k] \end{matrix} = \frac{1}{2} k \begin{matrix} ab \\ [k] \end{matrix}, \quad \tilde{S}^{ab} \begin{matrix} ab \\ (k) \end{matrix} = \frac{1}{2} k \begin{matrix} ab \\ (k) \end{matrix}, \quad \tilde{S}^{ab} \begin{matrix} ab \\ [k] \end{matrix} = -\frac{1}{2} k \begin{matrix} ab \\ [k] \end{matrix}. \quad (\text{A.2})$$

One recognizes that  $\gamma^a$  transform  $\begin{matrix} ab \\ (k) \end{matrix}$  into  $[-k]$ , never to  $\begin{matrix} ab \\ [k] \end{matrix}$ , while  $\tilde{\gamma}^a$  transform  $\begin{matrix} ab \\ (k) \end{matrix}$  into  $\begin{matrix} ab \\ [k] \end{matrix}$ ,

never to  ${}^{ab}[-k]$

$$\begin{aligned}\gamma^a({}^{ab}k) &= \eta^{aa}({}^{ab}[-k]), \quad \gamma^b({}^{ab}k) = -ik({}^{ab}[-k]), \quad \gamma^a({}^{ab}[k]) = (-k), \quad \gamma^b({}^{ab}[k]) = -ik\eta^{aa}({}^{ab}[-k]), \\ \tilde{\gamma}^a({}^{ab}k) &= -i\eta^{aa}({}^{ab}[k]), \quad \tilde{\gamma}^b({}^{ab}k) = -k({}^{ab}[k]), \quad \tilde{\gamma}^a({}^{ab}[k]) = i({}^{ab}k), \quad \tilde{\gamma}^b({}^{ab}[k]) = -k\eta^{aa}({}^{ab}k).\end{aligned}\quad (\text{A.3})$$

Let us add some useful relations

$$\begin{aligned}({}^{ab}k)({}^{ab}k) &= 0, \quad ({}^{ab}k)(-k) = \eta^{aa}({}^{ab}[k]), \quad [k][k] = [k], \quad [k](-k) = 0, \\ ({}^{ab}k)[k] &= 0, \quad [k]({}^{ab}k) = ({}^{ab}k), \quad (-k)[k] = (-k), \quad [k](-k) = 0.\end{aligned}\quad (\text{A.4})$$

Defining

$$(\pm i) = \frac{1}{2}(\tilde{\gamma}^a \mp \tilde{\gamma}^b), \quad (\pm 1) = \frac{1}{2}(\tilde{\gamma}^a \pm i\tilde{\gamma}^b), \quad (\text{A.5})$$

it follows

$$(\tilde{k})({}^{ab}k) = 0, \quad (\tilde{k})({}^{ab}k) = -i\eta^{aa}({}^{ab}[k]), \quad (\tilde{k})[k] = i({}^{ab}k), \quad (\tilde{k})[-k] = 0. \quad (\text{A.6})$$

We define the vacuum  $|\psi_0\rangle$  so that  $\langle ({}^{ab}k)({}^{ab}k) \rangle = 1$  and  $\langle [k][k] \rangle = 1$ .

Making a choice of the Cartan subalgebra set of the algebra  $S^{ab}$  and  $\tilde{S}^{ab}$  ( $S^{03}, S^{12}, S^{56}, S^{78}, \dots$ ) and ( $\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \tilde{S}^{78}, \dots$ ) an eigen state of all the members of the Cartan subalgebra, representing a weak chargeless  $u_R$ -quark with spin up, hyper charge (2/3) and colour (1/2, 1/(2√3)), for example, can be written as  $(+i)(+) \mid (+)(+) \mid (+)(-)(-)\mid \psi\rangle = \frac{1}{2^7}(\gamma^0 - \gamma^3)(\gamma^1 + i\gamma^2) |(\gamma^5 + i\gamma^6)(\gamma^7 + i\gamma^8) |(\gamma^9 + i\gamma^{10})(\gamma^{11} - i\gamma^{12})(\gamma^{13} - i\gamma^{14})|\psi\rangle$ . This state is an eigen state of all  $S^{ab}$  and  $\tilde{S}^{ab}$  which are members of the Cartan subalgebra. The definition of the charges can be found in the ref. [2, 3].

In Table II the eightplet of quarks of a particular colour charge ( $\tau^{33} = 1/2, \tau^{38} = 1/(2\sqrt{3})$ ) and the  $U(1)_{II}$  charge ( $\tau^4 = 1/6$ ) is presented in our technique [9, 10], as products of nilpotents and projectors. The operators  $\tilde{S}^{ab}$  generate families from the starting  $u_R$  quark, transforming  $u_R$  quark from Table (I) to the  $u_R$  of another family, keeping all the properties with respect to  $S^{ab}$  unchanged. The eight families of the first member of the eightplet of quarks from Table II, for example, that is of the right handed  $u_R^c$ -quark with spin  $\frac{1}{2}$ , are presented in the left column of Table III. The eight-plet of the corresponding right handed neutrinos with spin up is presented in the right column of the same table. All the other

i		$ \psi_i\rangle$	$\Gamma^{(1,3)}$	$S^{12}$	$\tau^{13}$	$Y$	$Q$
		Octet of quarks					
1	$u_R^{c1}$	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ (+i) & (+) &   & (+) & (+) &    & (+) & [-] & [-] \end{smallmatrix}$	1	$\frac{1}{2}$	0	$\frac{2}{3}$	$\frac{2}{3}$
2	$u_R^{c1}$	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ [-i] & [-] &   & (+) & (+) &    & (+) & [-] & [-] \end{smallmatrix}$	1	$-\frac{1}{2}$	0	$\frac{2}{3}$	$\frac{2}{3}$
3	$d_R^{c1}$	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ (+i) & (+) &   & [-] & [-] &    & (+) & [-] & [-] \end{smallmatrix}$	1	$\frac{1}{2}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
4	$d_R^{c1}$	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ [-i] & [-] &   & [-] & [-] &    & (+) & [-] & [-] \end{smallmatrix}$	1	$-\frac{1}{2}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
5	$d_L^{c1}$	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ [-i] & (+) &   & [-] & (+) &    & (+) & [-] & [-] \end{smallmatrix}$	-1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$
6	$d_L^{c1}$	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ (+i) & [-] &   & [-] & (+) &    & (+) & [-] & [-] \end{smallmatrix}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$
7	$u_L^{c1}$	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ [-i] & (+) &   & (+) & [-] &    & (+) & [-] & [-] \end{smallmatrix}$	-1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$
8	$u_L^{c1}$	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ (+i) & [-] &   & (+) & [-] &    & (+) & [-] & [-] \end{smallmatrix}$	-1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$

TABLE II: The 8-plet of quarks - the members of  $SO(1,7)$  subgroup of the group  $SO(1,13)$ , belonging to one Weyl spinor representation of  $SO(1,13)$  is presented in the technique [10]. It contains the left handed weak charged quarks and the right handed weak chargeless quarks of a particular colour  $(1/2, 1/(2\sqrt{3}))$ . Here  $\Gamma^{(1,3)}$  defines the handedness in  $(1+3)$  space,  $S^{12}$  defines the ordinary spin,  $\tau^{13}$  defines the third component of the weak charge,  $Y$  is the hyper charge,  $Q = Y + \tau^{13}$  is the electromagnetic charge. The vacuum state  $|\psi_0\rangle$ , on which the nilpotents and projectors operate, is not shown. The basis is the massless one. One easily sees that  $\gamma^0 \begin{smallmatrix} 78 \\ (-) \end{smallmatrix}$  transforms the first line ( $u_R^{c1}$ ) into the seventh one ( $u_L^{c1}$ ), and  $\gamma^0 \begin{smallmatrix} 78 \\ (+) \end{smallmatrix}$  transforms the third line ( $d_R^{c1}$ ) into the fifth one ( $d_L^{c1}$ ).

members of any of the eight families of quarks or leptons follow from any member of a particular family by the application of the operators  $S^{ab}$  on this particular member.

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$I_R$	$u_R^{c1}$	$\begin{array}{c} 03 \quad 12 \quad 56 \quad 78 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \\ (+i) \quad (+) \mid (+) \quad (+) \parallel (+) \quad [-] \quad [-] \end{array}$	$\nu_R$	$\begin{array}{c} 03 \quad 12 \quad 56 \quad 78 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \\ [+i] \quad (+) \mid (+) \quad (+) \parallel (+) \quad (+) \quad (+) \end{array}$
$II_R$	$u_R^{c1}$	$\begin{array}{c} 03 \quad 12 \quad 56 \quad 78 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \\ [+i] \quad (+) \mid (+) \quad (+) \parallel (+) \quad [-] \quad [-] \end{array}$	$\nu_R$	$\begin{array}{c} 03 \quad 12 \quad 56 \quad 78 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \\ (+i) \quad (+) \mid (+) \quad (+) \parallel (+) \quad (+) \quad (+) \end{array}$
$III_R$	$u_R^{c1}$	$\begin{array}{c} 03 \quad 12 \quad 56 \quad 78 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \\ (+i) \quad (+) \mid (+) \quad (+) \parallel (+) \quad [-] \quad [-] \end{array}$	$\nu_R$	$\begin{array}{c} 03 \quad 12 \quad 56 \quad 78 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \\ (+i) \quad (+) \mid (+) \quad (+) \parallel (+) \quad (+) \quad (+) \end{array}$
$IV_R$	$u_R^{c1}$	$\begin{array}{c} 03 \quad 12 \quad 56 \quad 78 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \\ [+i] \quad (+) \mid (+) \quad (+) \parallel (+) \quad [-] \quad [-] \end{array}$	$\nu_R$	$\begin{array}{c} 03 \quad 12 \quad 56 \quad 78 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \\ [+i] \quad (+) \mid (+) \quad (+) \parallel (+) \quad (+) \quad (+) \end{array}$

TABLE III: Four families of the right handed  $u_R^{c1}$  quark with spin  $\frac{1}{2}$ , the colour charge ( $c^1 = (\tau^{33} = 1/2, \tau^{38} = 1/(2\sqrt{3}))$ ), and of the colourless right handed neutrino  $\nu_R$  of spin  $\frac{1}{2}$  are presented in the left and in the right column, respectively. All the families follow from the starting one by the application of the operators  $\tilde{S}^{ab}$ ,  $a, b \in \{0, 1, 2, \dots, 8\}$ . The generators  $S^{ab}$ ,  $a, b \in \{0, 1, 2, \dots, 8\}$  transform equivalently the right handed neutrino  $\nu_R$  of spin  $\frac{1}{2}$  to all the colourless members of the same family.

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- [13] More about the two kinds of the Clifford algebra objects can be found in the refs. [1–3, 10].  
The appendix IV gives a short overview.
- [14] The two triplets  $(\vec{\tilde{A}}_{\mp}^{\tilde{N}_L}, \vec{\tilde{A}}_{\mp}^1)$  and the singlets  $(A_{\mp}^Q, A_{\mp}^{Q'}, A_{\mp}^{Y'})$  appear in the mass term (Eq.(1)) contributing to masses of fermions since they are assumed to have nonzero vacuum expectation values. Since  $\tau^{1i} \psi_R^\alpha = 0$ , the  $\vec{\tilde{A}}_{\mp}^1$  and  $\vec{\tilde{A}}_{\mp}^o$  triplets do not contribute to the masses of fermions on the tree level.
- [15] Let me here refer to the simple case of subsect. II A by paying attention to the reader that in Table I the two vacuum expectation values of each of the two scalar triplets,  $(\tilde{a}^{\tilde{N}_L\mp}, \tilde{a}^{\tilde{N}_L3})$  and  $(\tilde{a}^{1\mp}, \tilde{a}^{13})$ , are expected to have the property  $(\tilde{a}^{\tilde{N}_L+} \approx \tilde{a}^{\tilde{N}_L-})$  and  $(\tilde{a}^{1+} \approx \tilde{a}^{1-})$ , respectively, or at least very close to this. Then superposition of the scalar fields, to which different families couple, might differ a lot.
- [16] Let us also recognize that to come from the *spin-charge-family* theory to the *standard model* not only must all the scalar fields originated in the scalar components of the vielbeins and the two kinds of the spin connection fields be replaced by one scalar field, which is a weak doublet, but must the effect of these scalar fields on the fermion properties be replaced by their measured properties.